Using Derivative Analysis to Improve Pumping Test Interpretation with the Cooper and Jacob Method

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What Is a Pumping Test?

- An aquifer test performed with a controlled pumping rate
  - constant-rate test
  - step-drawdown test (well performance)
  - recovery test
- Water-level response (drawdown) measured in control well and one or more observation wells

What Are the Objectives of a Pumping Test?

- Estimation of hydraulic properties (aquifers and aquitards)
- Detection of boundaries
- Evaluation of well performance (well loss)

Analysis of Pumping Test Data

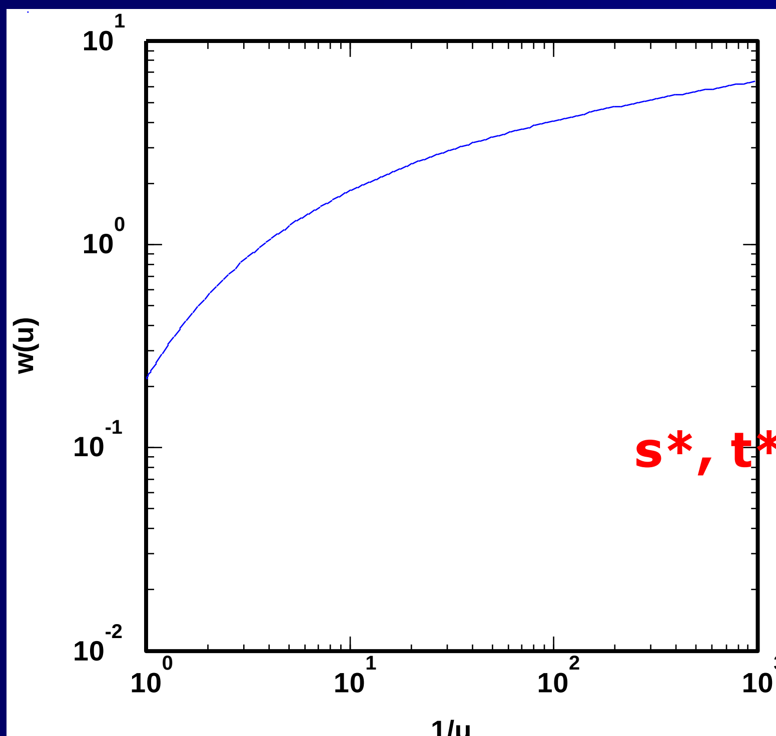
Traditional Methods

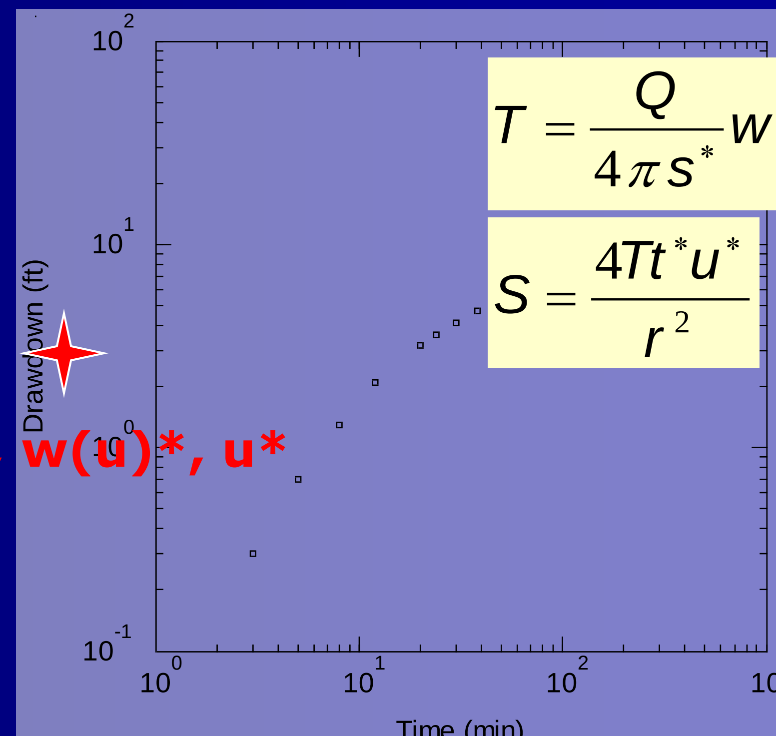
In The Beginning…

there was Theis!

- Theis (1935) introduced a type-curve matching technique for estimating aquifer properties from a constant-rate pumping test assuming a fully penetrating pumping well in a homogeneous and isotropic nonleaky confined aquifer of infinite extent and constant thickness…

In The Beginning…
there was Theis!





$$T = \frac{Q}{4\pi s^*} w(u)^*$$

$$S = \frac{4Tt^* u^*}{r^2}$$

s*, t*, w(u)*, u*

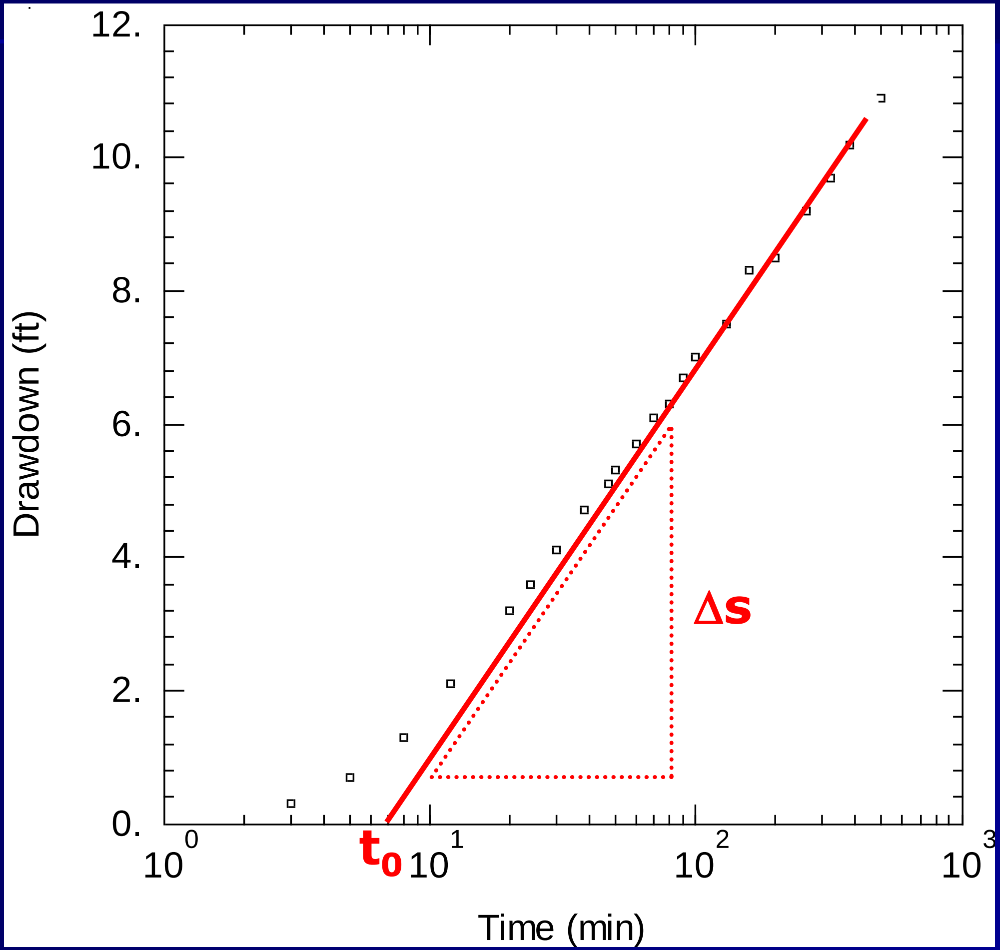
And Then Came…

Cooper and Jacob!

- Cooper and Jacob (1946) subsequently discovered that the Theis solution, drawn on semilog axes, plots as a straight line after sufficiently long periods of pumping…

And Then Came…

Cooper-Jacob!



$$T = \frac{2.303Q}{4\pi \Delta s}$$

$$S = \frac{2.25Tt_0}{r^2}$$

valid when $u \leq 0.01$ (i.e., when slope is constant) where $u = r^2S/4Tt$

Pumping Test Data Analysis

- How often is the Cooper and Jacob method the first step in your analysis of pumping test data?
- Are there techniques you could use to get more reliable results?

A Different Approach…

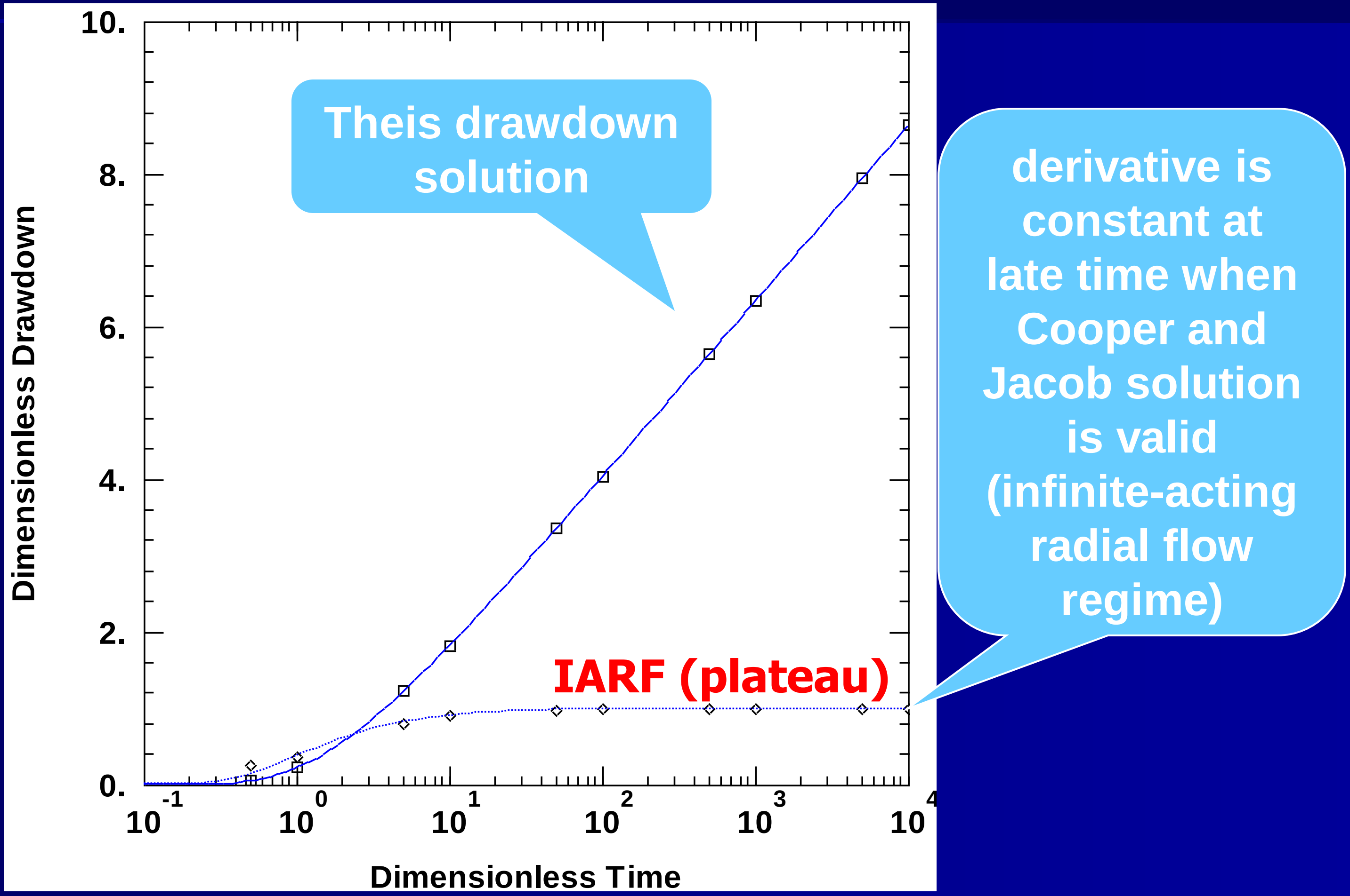
- A more productive approach to pumping test data analysis begins with the application of ***derivative analysis*** that helps you to:
  - identify common flow regimes
  - guide subsequent curve matching
- What is **derivative analysis**?

Derivative Analysis

- Technique popularized in the petroleum industry (Bourdet et al. 1983)
- Plot of $\partial s / \partial \ln t$ vs t
- Derivatives are calculated from field data
- A ***derivative plot***, which combines the display of drawdown and derivative data, is a powerful diagnostic and curve matching tool

Interpretation of Derivative

slope of drawdown data on semilog plot



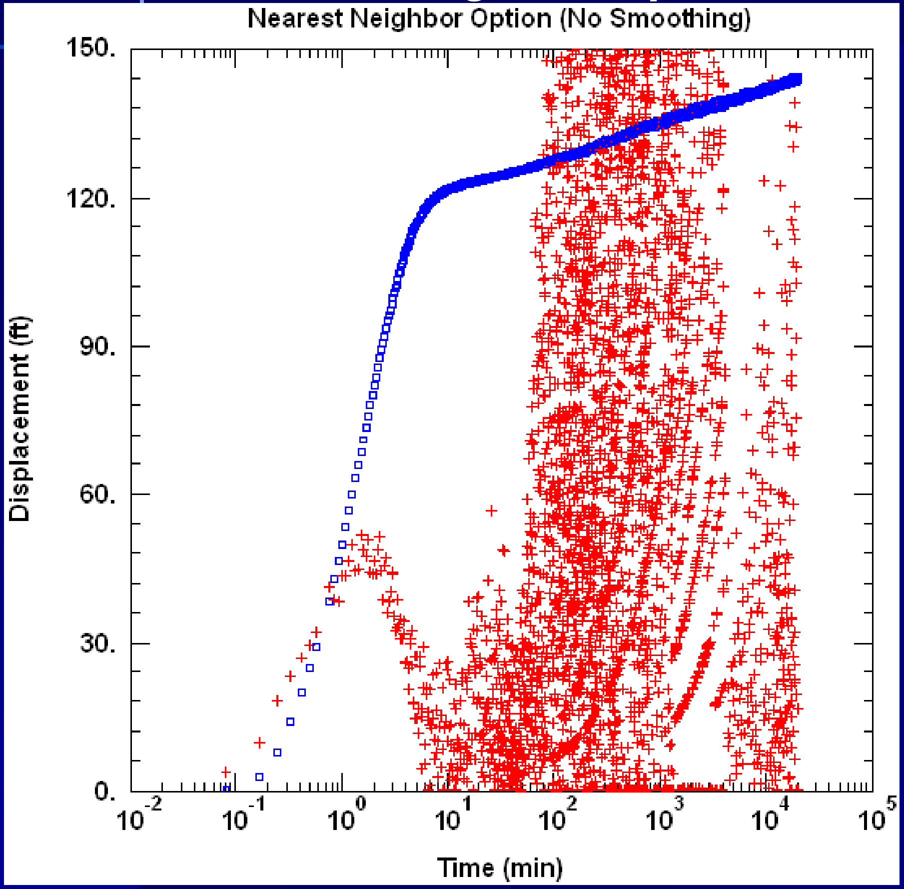
Derivative Smoothing

- Derivatives computed directly from field data are often ***noisy***
- ***Four smoothing options*** are available in AQTESOLV to reduce noise
  - ***nearest neighbor*** (no smoothing)
  - ***Bourdet*** **method**
  - ***Spane*** **method**
  - ***smoothing***

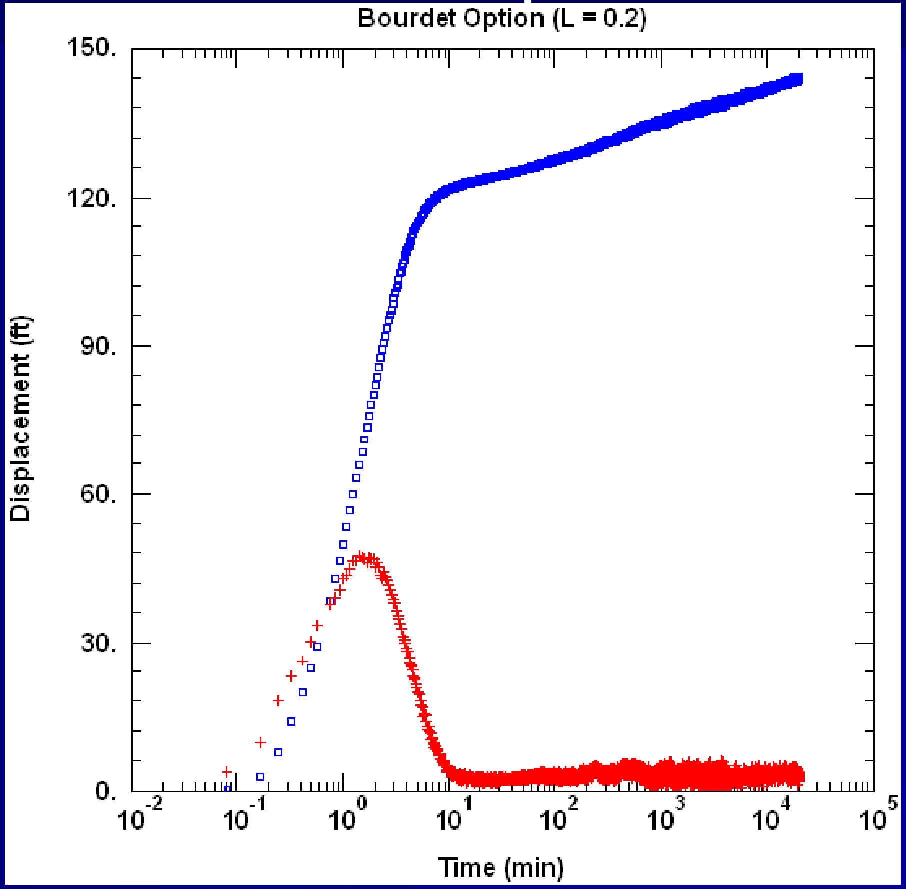
Begin with nearest neighbor method. Avoid excessive smoothing!

Effect of Smoothing

Nearest Neighbor Option



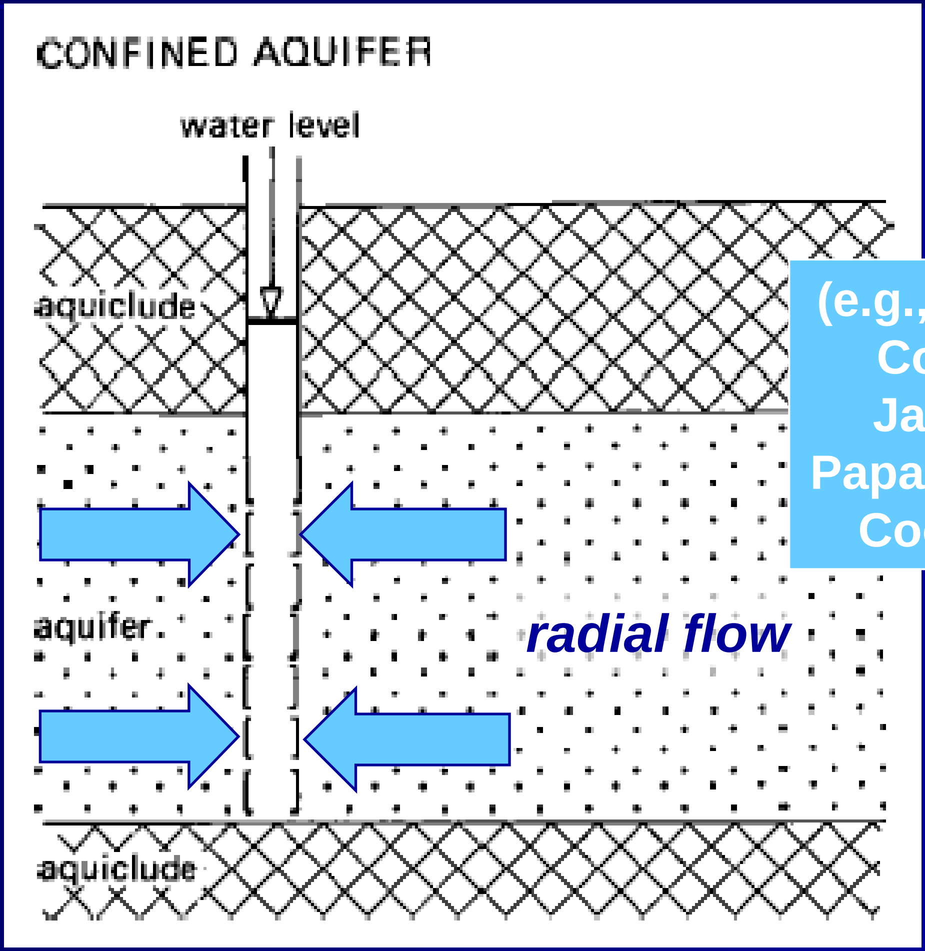
Bourdet Option



Simple Example: Single-Well Test

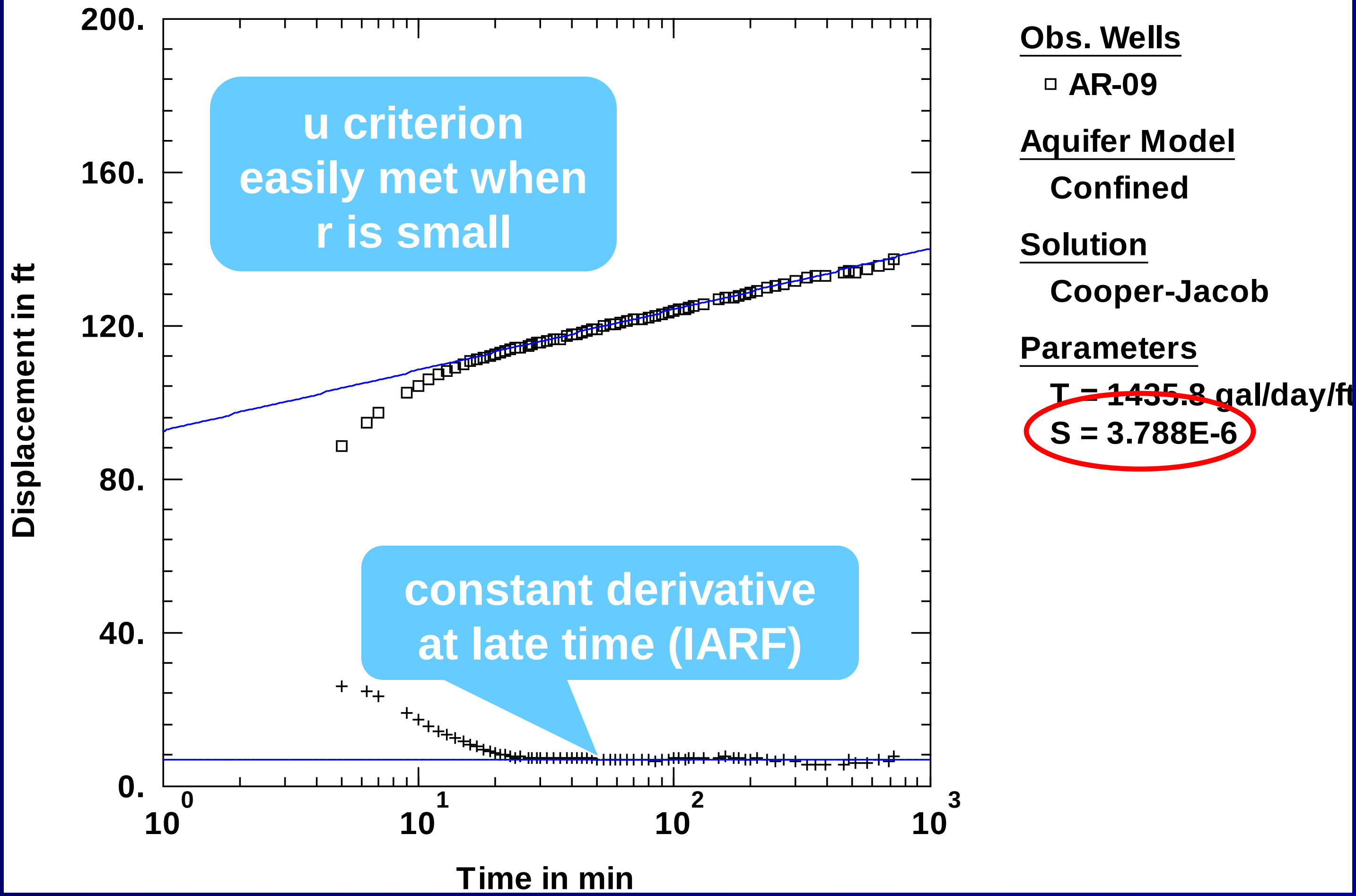
- 12-hour constant-rate test (Q = 86.9 gpm)
- Recovery monitored for 1 hour
- Aquifer–fractured bedrock (Triassic sandstone, siltstone, shale sequence)
- Upper boundary–water table
- Lower boundary–unknown (total depth of well is 465 ft)
- Assume test well is fully penetrating but most of water may be coming from lower 150 ft
- $r_c = r_w = 4$ inches

Nonleaky Confined Aquifer

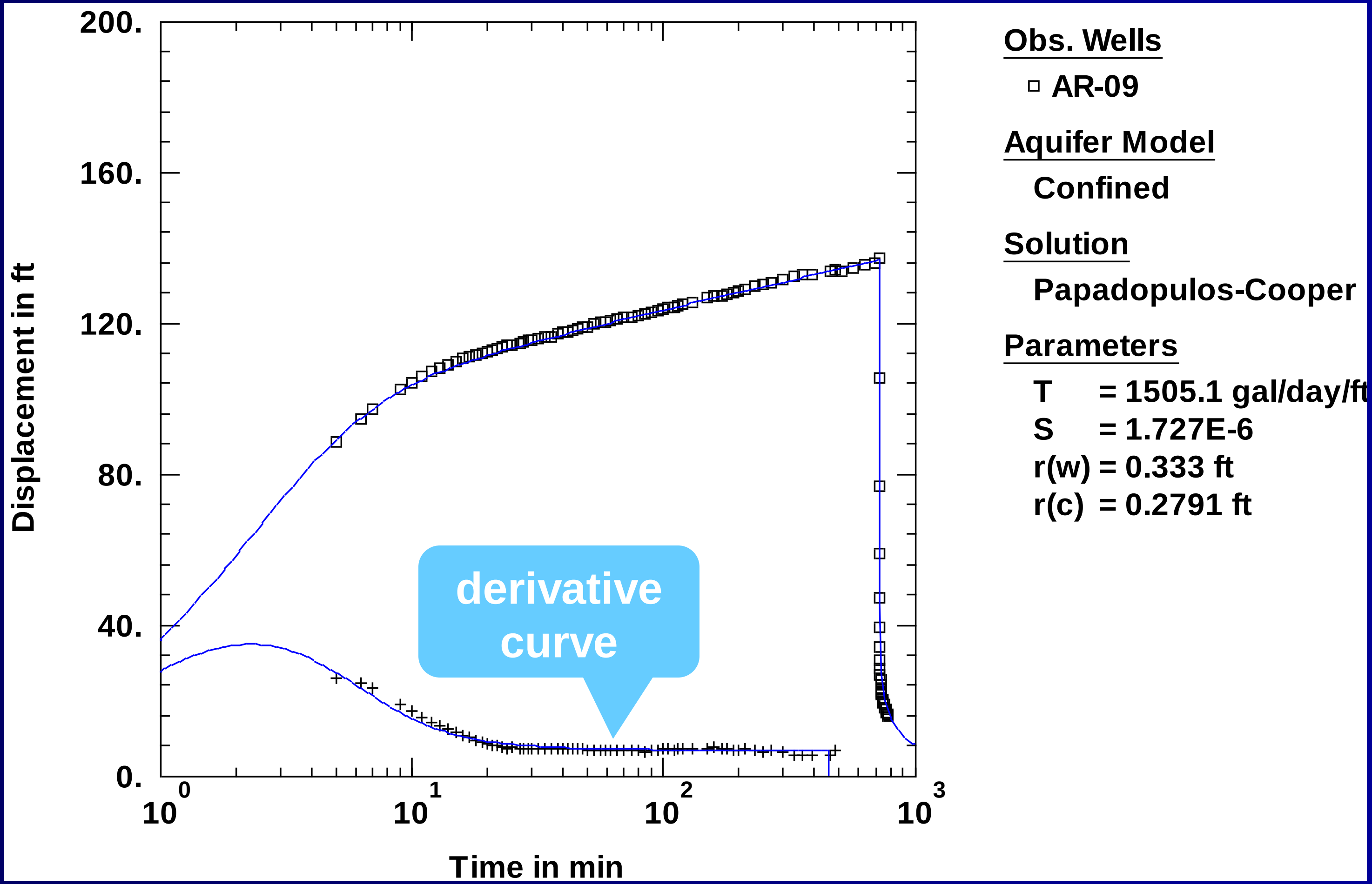


(e.g., Theis 1935; Cooper and Jacob 1946; Papadopulos and Cooper 1967)

Cooper and Jacob Match



Papadopulos and Cooper Match



Key Concepts and Tips

- Combine derivative analysis with the Cooper and Jacob method to
  - identify IARF period (derivative plateau)
  - improve fitting of straight line
- Cooper and Jacob can obtain results comparable with more rigorous methods with less effort

Key Concepts and Tips

- Cooper and Jacob applied to single-well tests can yield reliable estimates of T; however, S often will be biased due to partial penetration and/or well losses.

Case Study: Coastal Aquifer
Oude Korendijk, The Netherlands

- 14-hour constant-rate test (Q = 788 m^3/day)
- Aquifer–7 m of coarse sand with some gravel
- Upper boundary–18 m of clay, peat and clayey fine sand; note clayey fine sand directly above aquifer
- Lower boundary–fine sand and clay sediments
- Test well is fully penetrating
- Observation wells at r = 30, 90 and 215 m from pumped well

Source: Kruseman and de Ridder (1994)

Stratigraphy

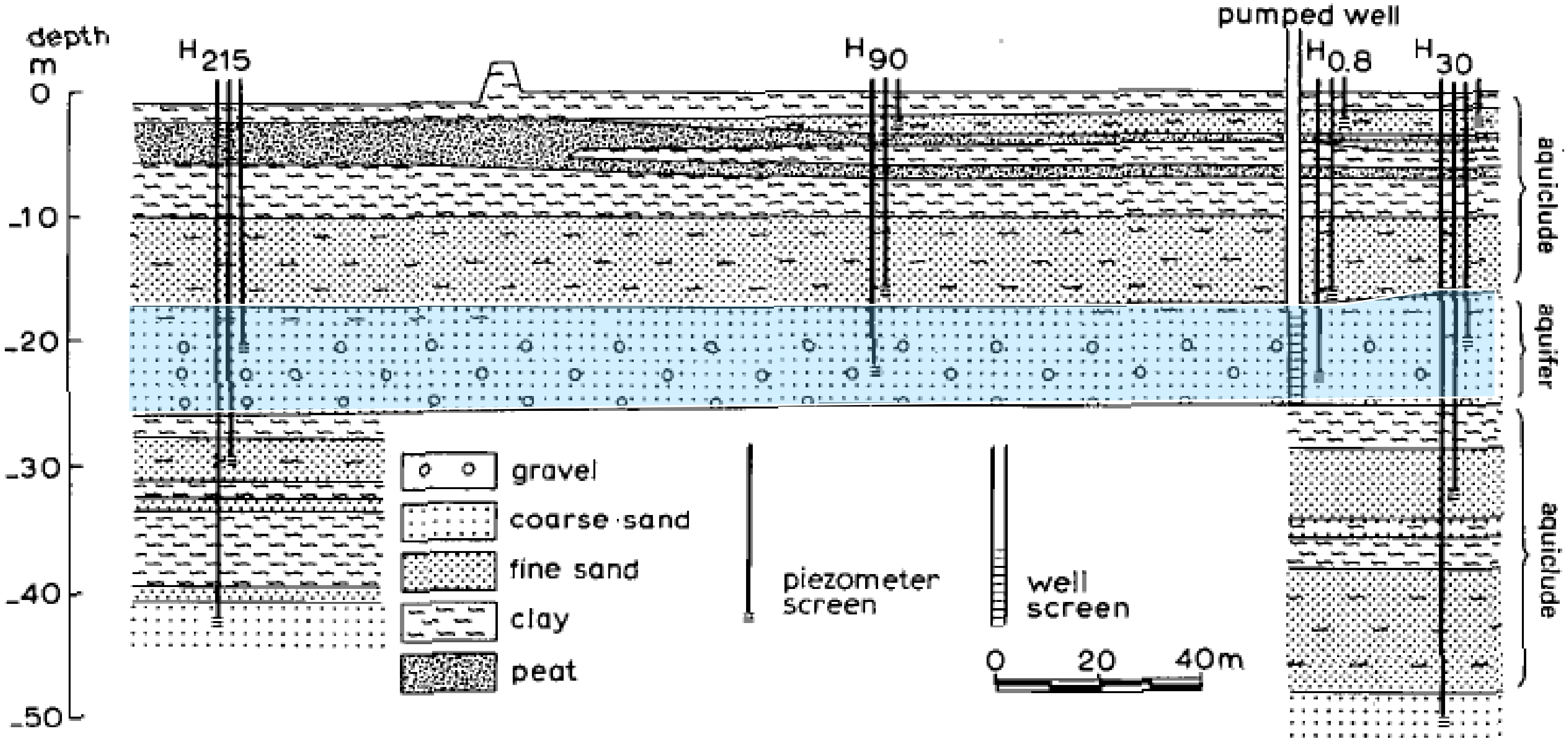


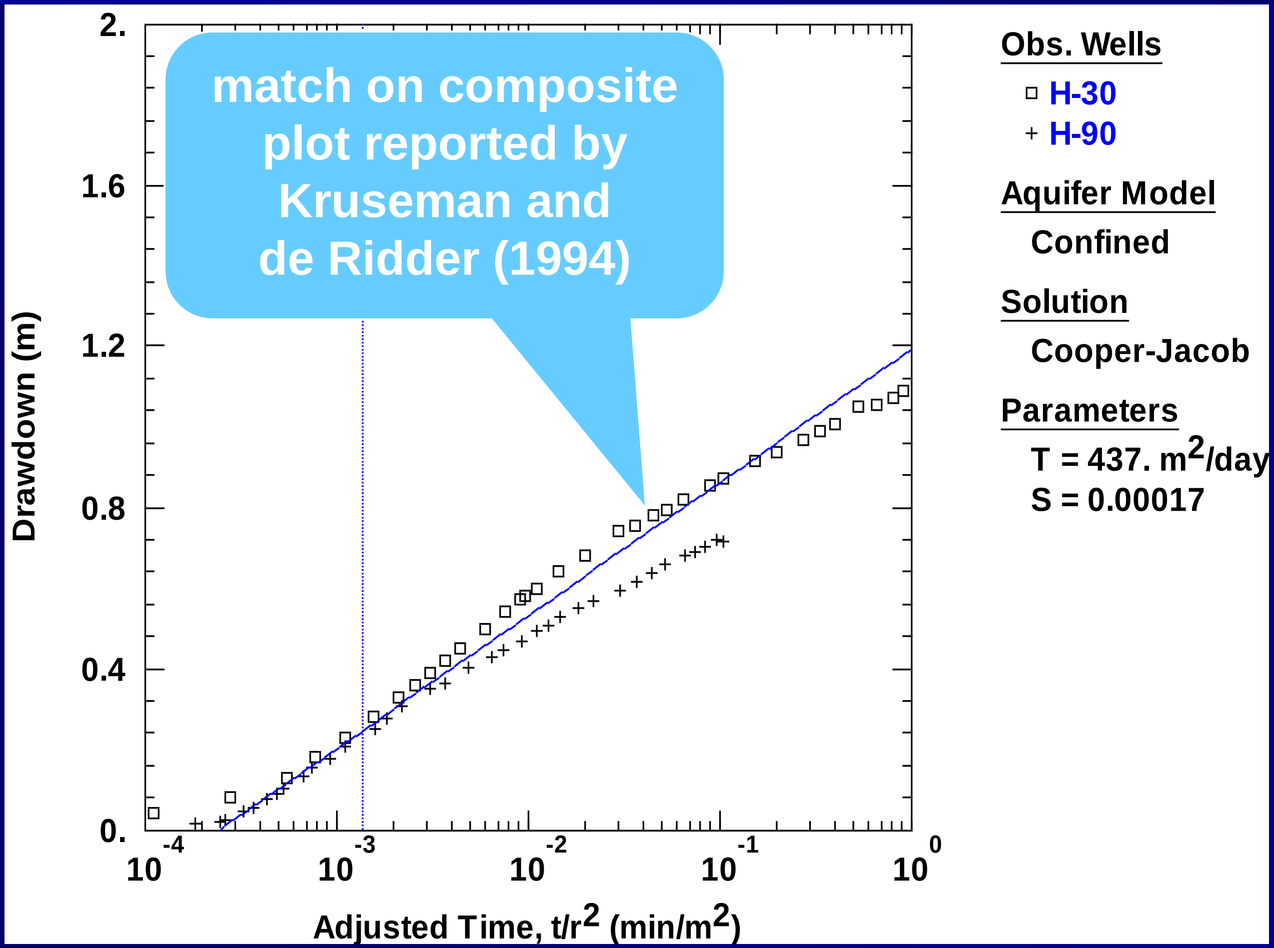
Figure 3.2 Lithological cross-section of the pumping-test site 'Oude Korendijk', The Netherlands (after Wit 1963)

from Kruseman and de Ridder (1994)

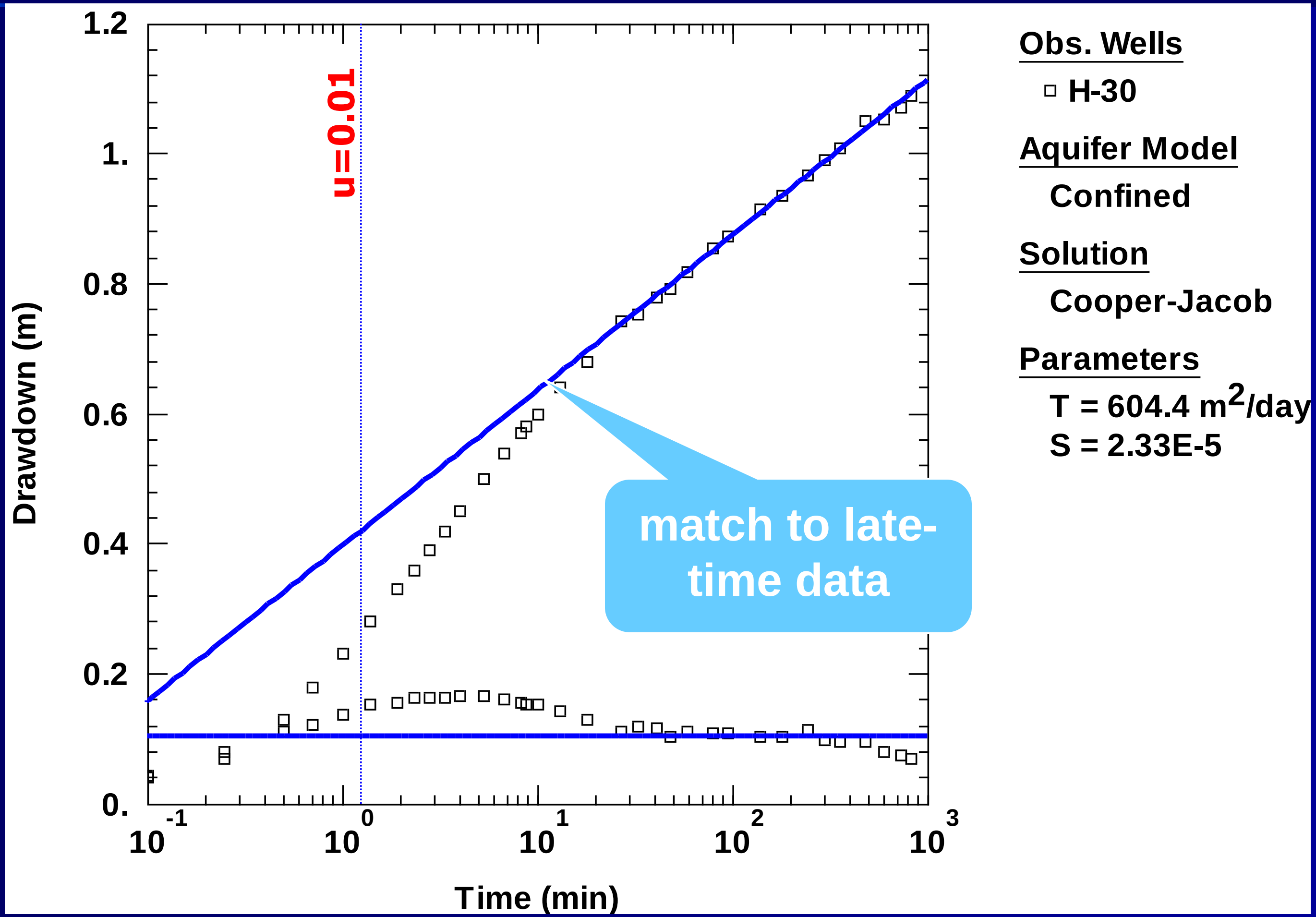
Cooper and Jacob Analysis

- Kruseman and de Ridder assumed a nonleaky confined aquifer for the analysis of the constant-rate pumping test.
- Let's consider interpretations of drawdown data with and without derivative analysis…

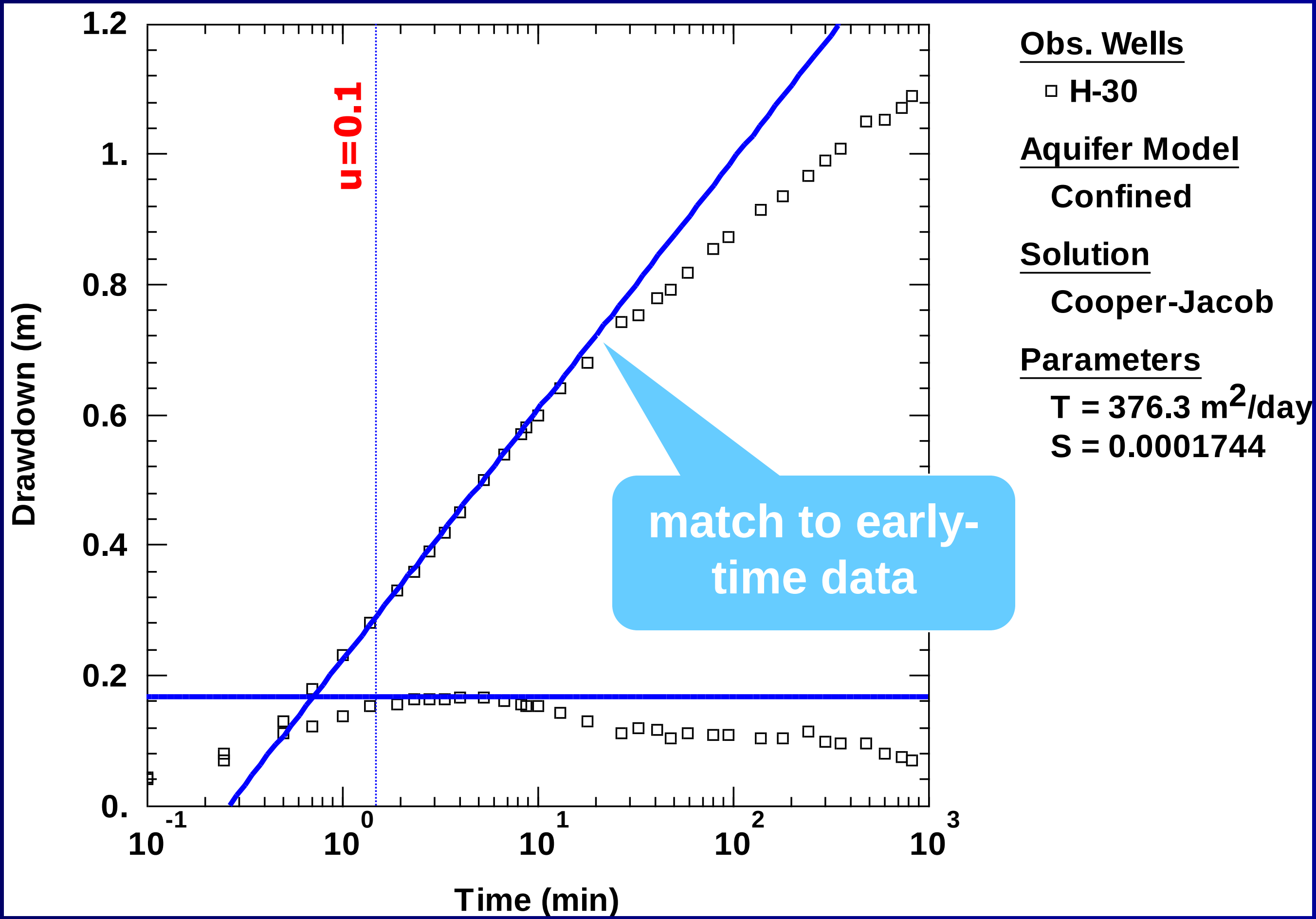
Cooper and Jacob, r=30 and 90 m



Cooper and Jacob, r=30 m, late



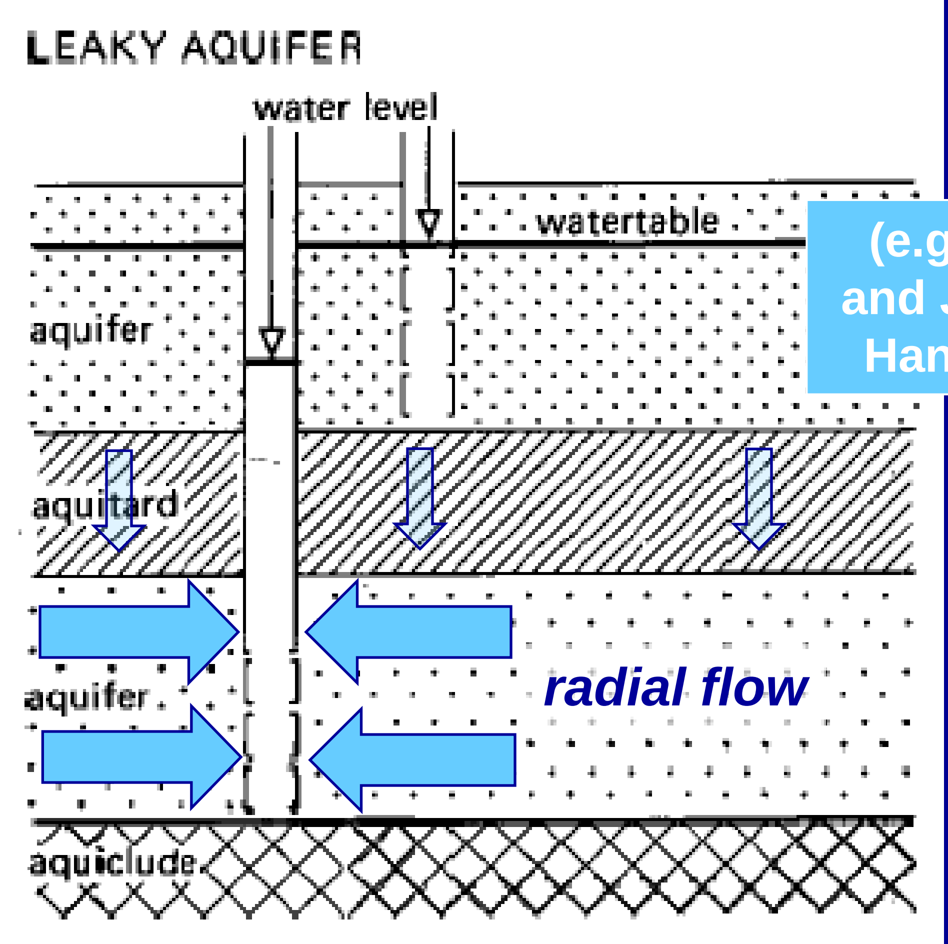
Cooper and Jacob, r=30 m, early



Cooper and Jacob Results

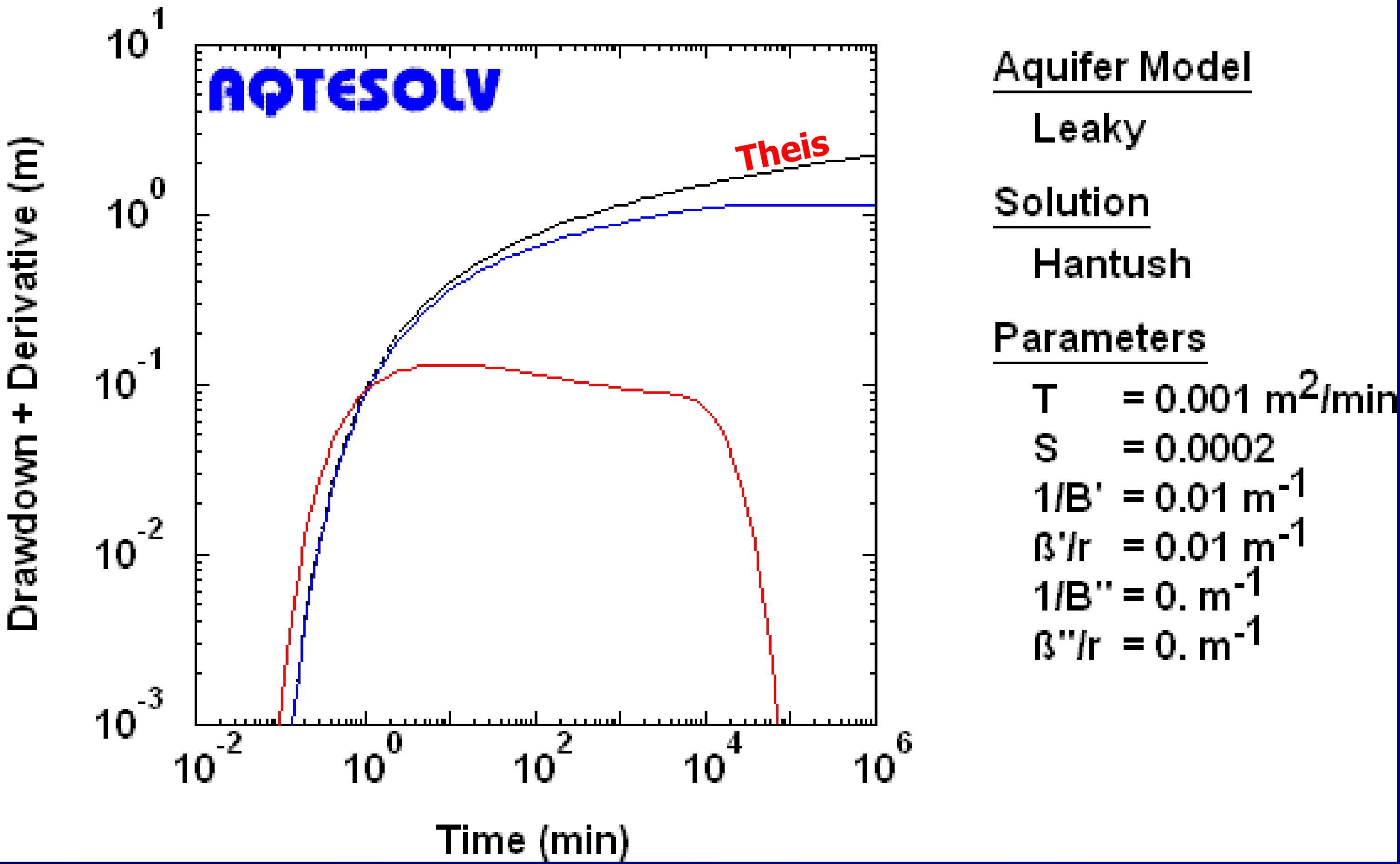
- We have three very different estimates of T and S. Which interpretation is most reliable?
- Let’s consider the response of a leaky confined aquifer with aquitard storage and its associated derivative plot…

Leaky Confined Aquifer

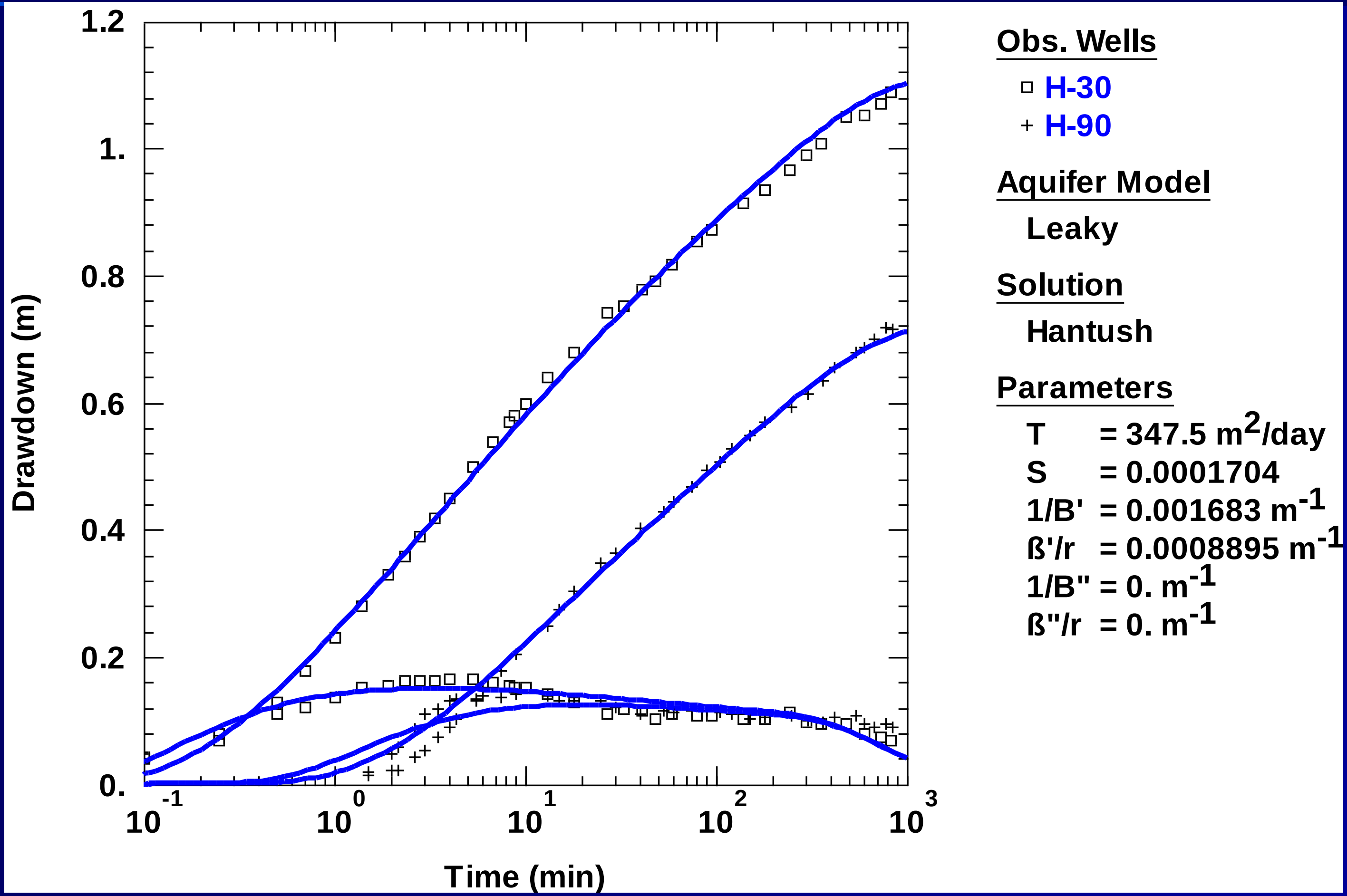


(e.g., Hantush and Jacob 1955; Hantush 1960)

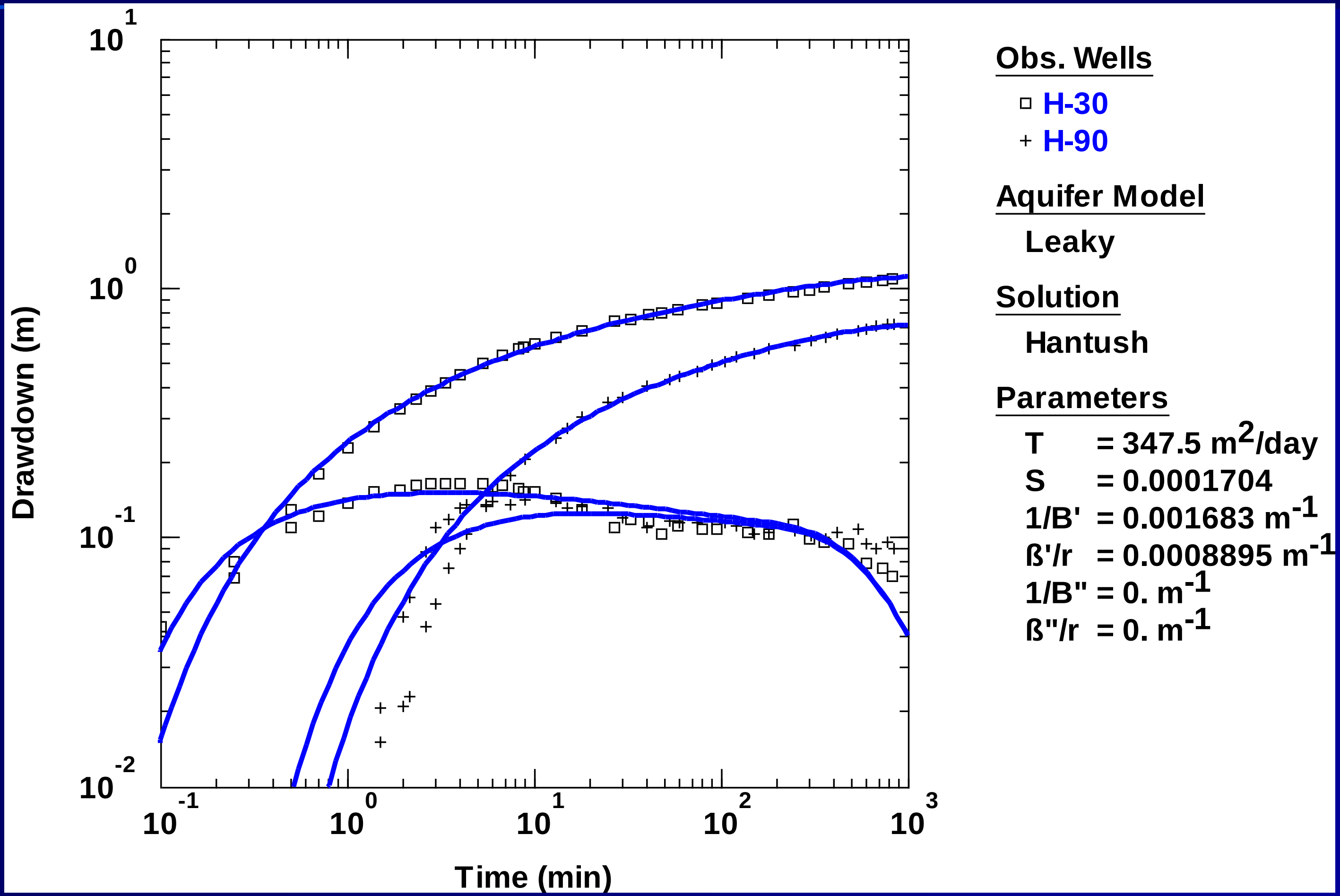
Derivative Plot, Leaky Confined Aquifer



Hantush, r=30 and 90 m, leaky aquitard



Hantush, r=30 and 90 m, leaky aquitard



Summary of Results

- Estimates of T (leaky confined):
  - ***348 m^2/day*** (compressible aquitard)
- Estimates of T (nonleaky confined):
  - ***375 m^2/day*** (Cooper-Jacob, early)
  - ***437 m^2/day*** (Cooper-Jacob, composite)
  - ***600 m^2/day*** (Cooper-Jacob, late)

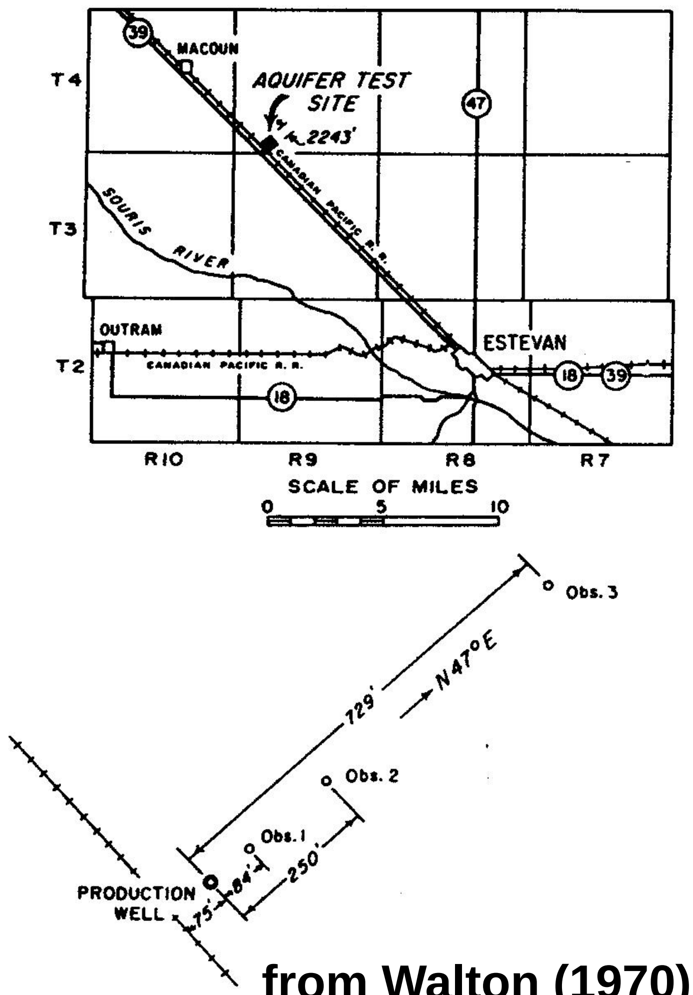
Key Concepts and Tips

- Apply the Cooper and Jacob method in conjunction with derivative analysis to provide reasonable preliminary estimates of T and S for leaky confined aquifers.
- Use derivative analysis to choose and refine conceptual model(s) of groundwater flow system.

Case Study: Channel Aquifer
Estevan, Saskatchewan

- Walton (1970) presented data and results from an eight-day pumping test conducted in a buried sand-and-gravel channel aquifer near Estevan, Saskatchewan, Canada
  - Q = 457 to 464 imperial gallons-per-minute
  - b = 30 to 90 ft (typical)
  - width of channel = 3,000 to 12,000 ft (typical)

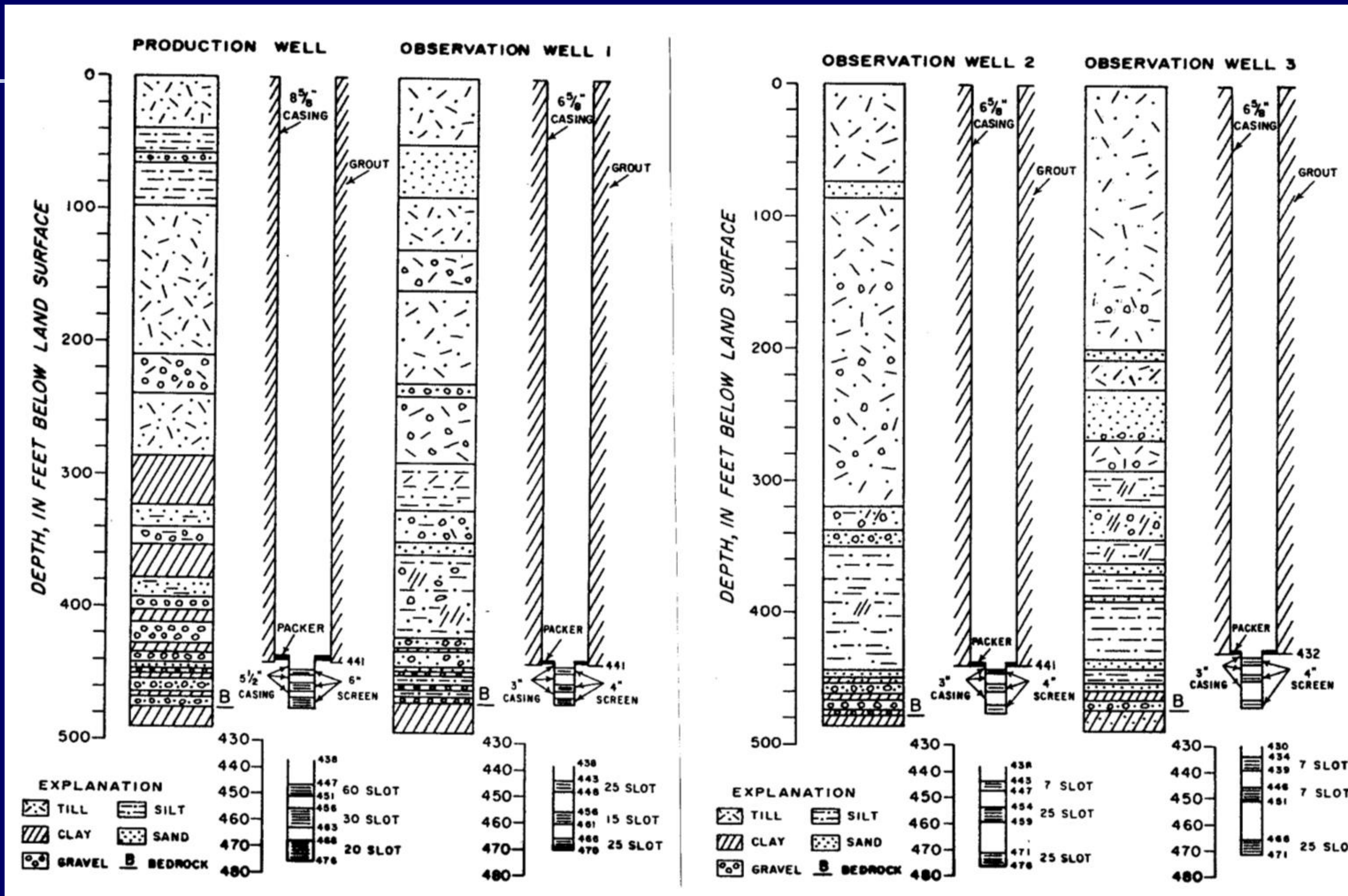
Well Locations



from Walton (1970)

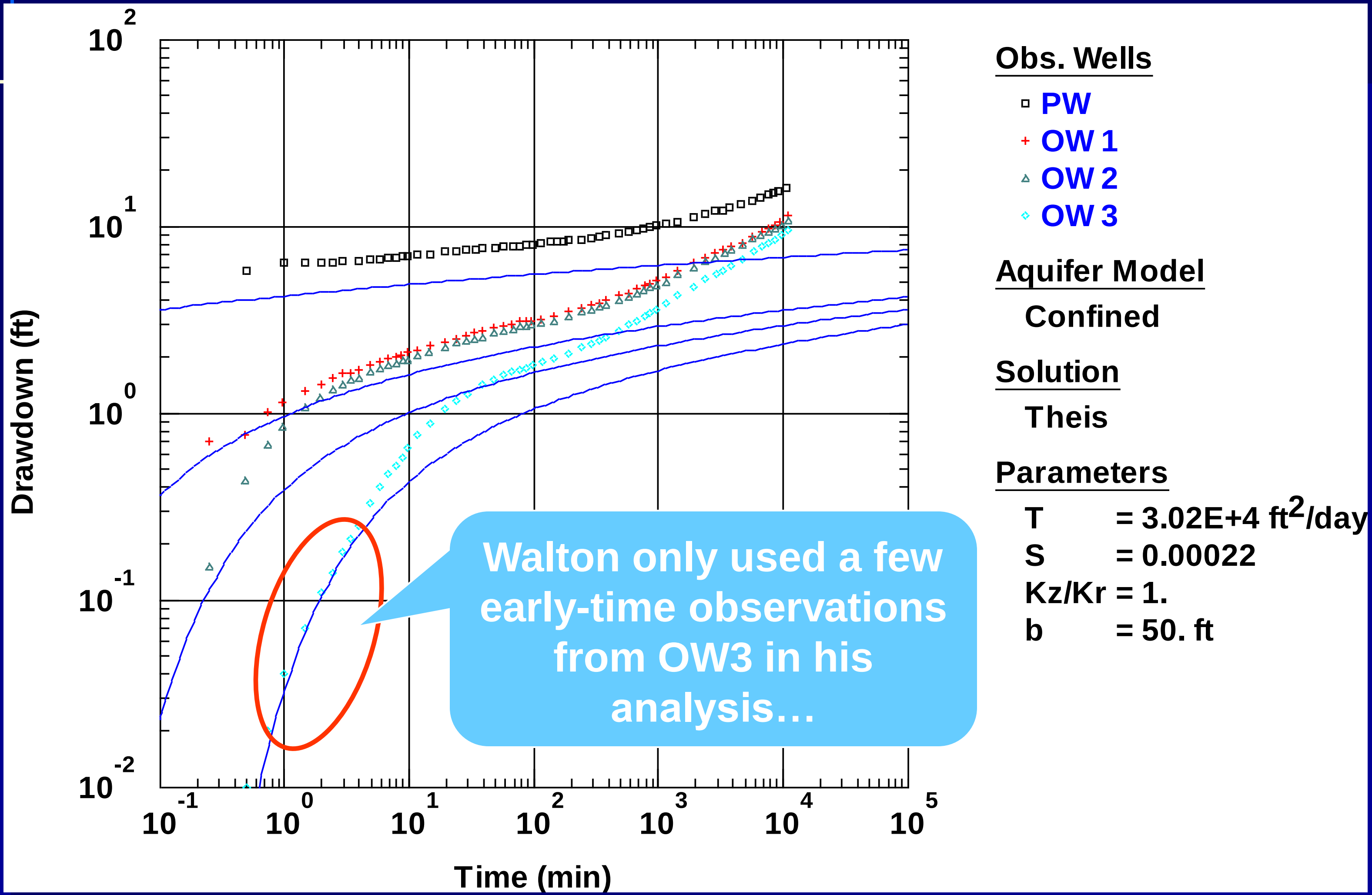
- three observation wells
  - r = 84 ft
  - r = 250 ft
  - r = 729 ft

Well Logs

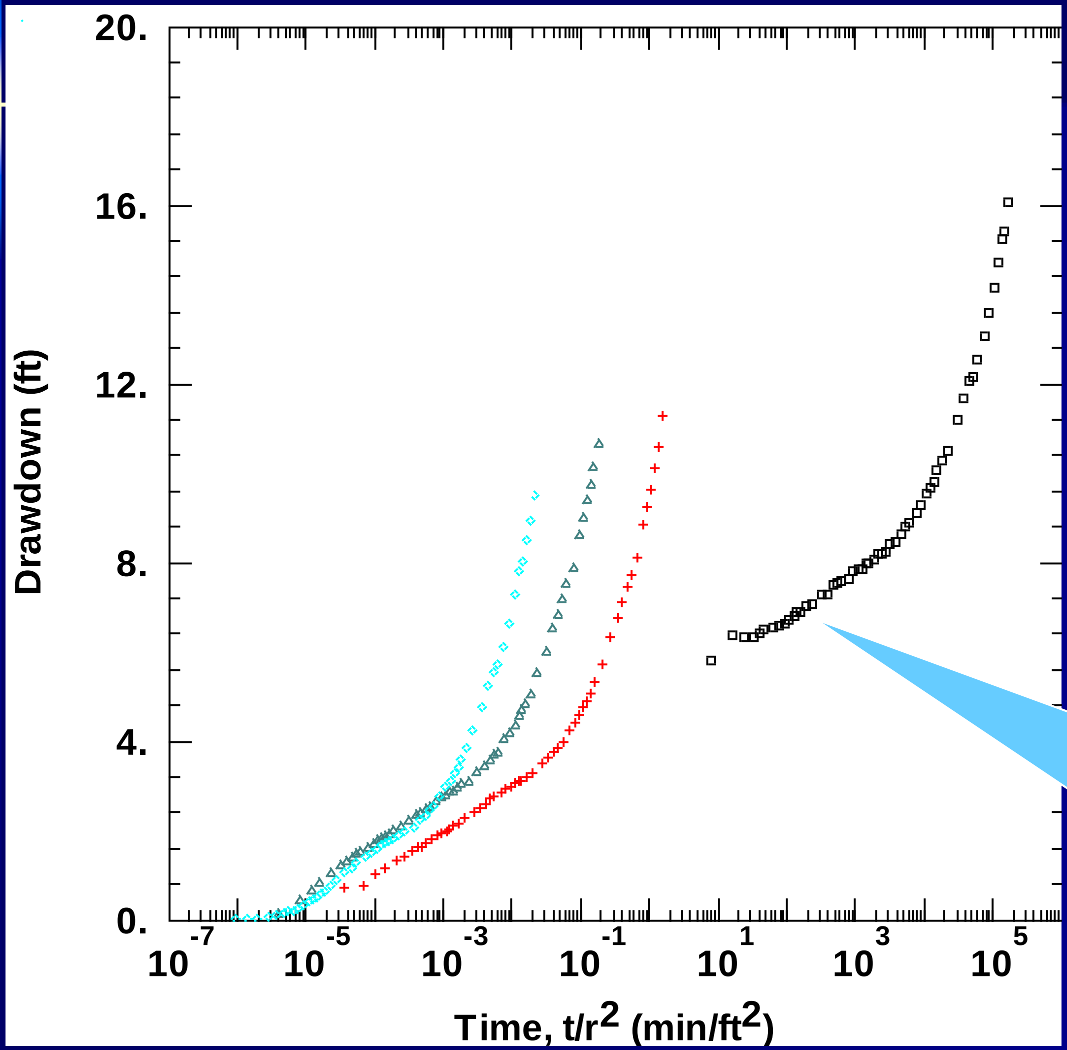


from Walton (1970)

Walton’s Analysis

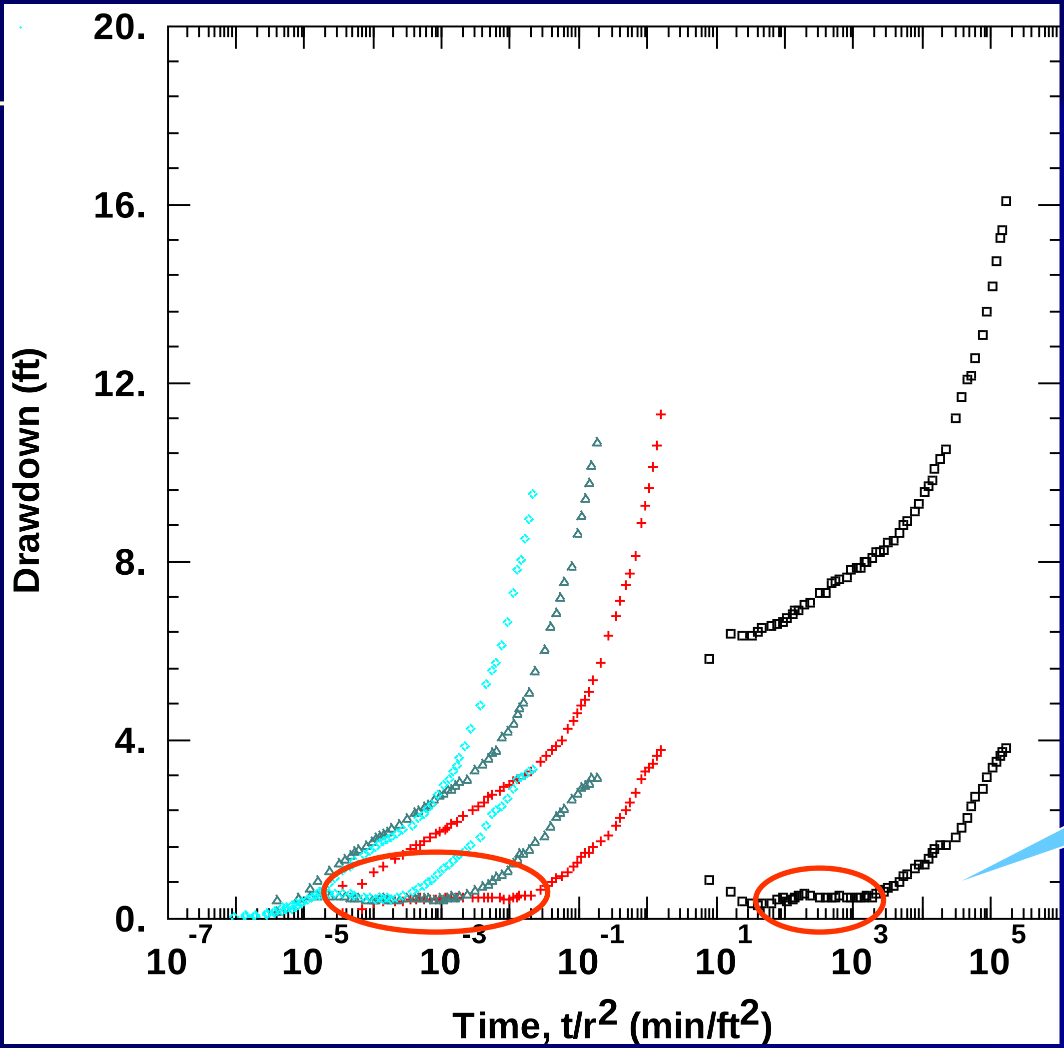


Composite Plot



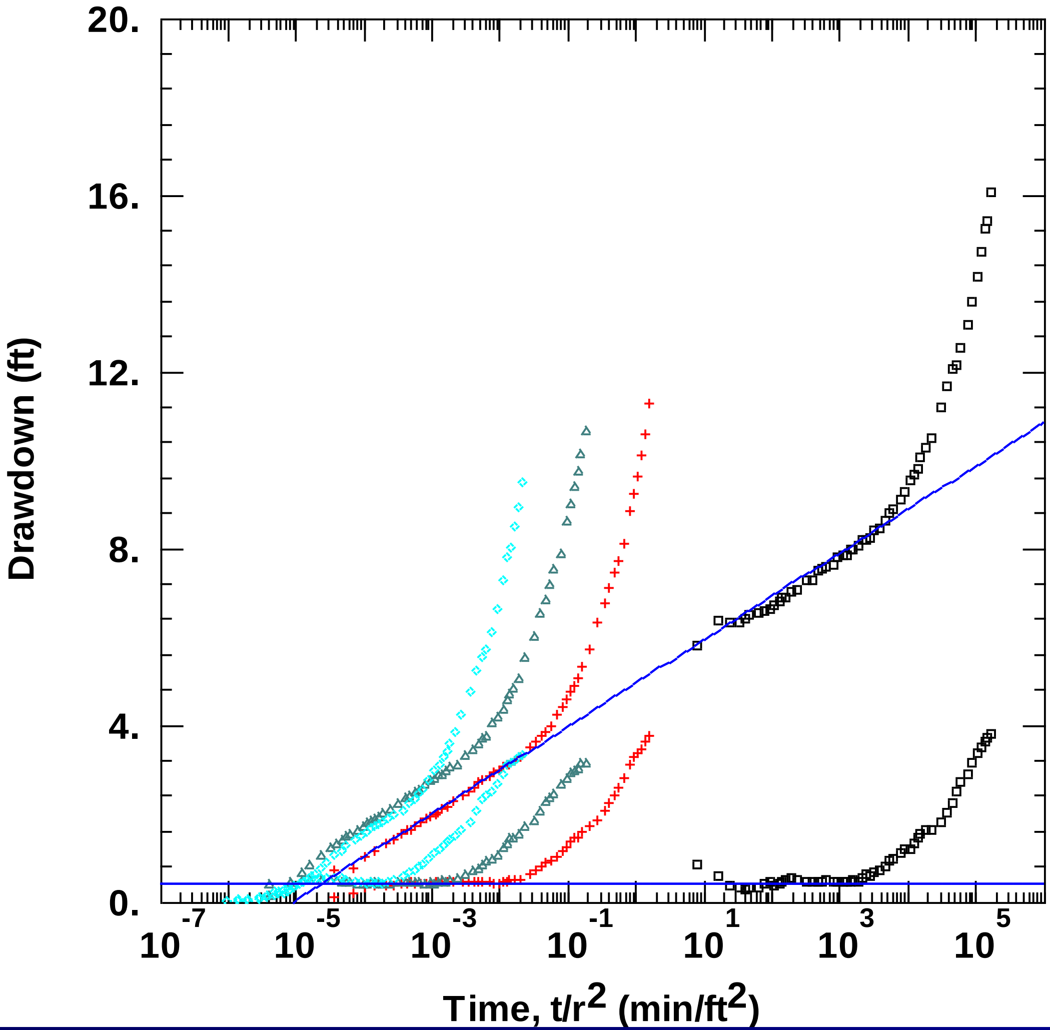
all wells show same slope at intermediate time suggesting infinite-acting aquifer conditions

Derivative Plot



derivative plot confirms constant slope during infinite-acting period

Cooper and Jacob Match



Obs. Wells

- PW
- OW 1
- OW 2
- OW 3

Aquifer Model

Confined

Solution

Cooper-Jacob

Parameters

T = 1.983E+4 ft^2/day
S = 0.0002836

aquifer properties determined from infinite-acting period

Image Well Arrays

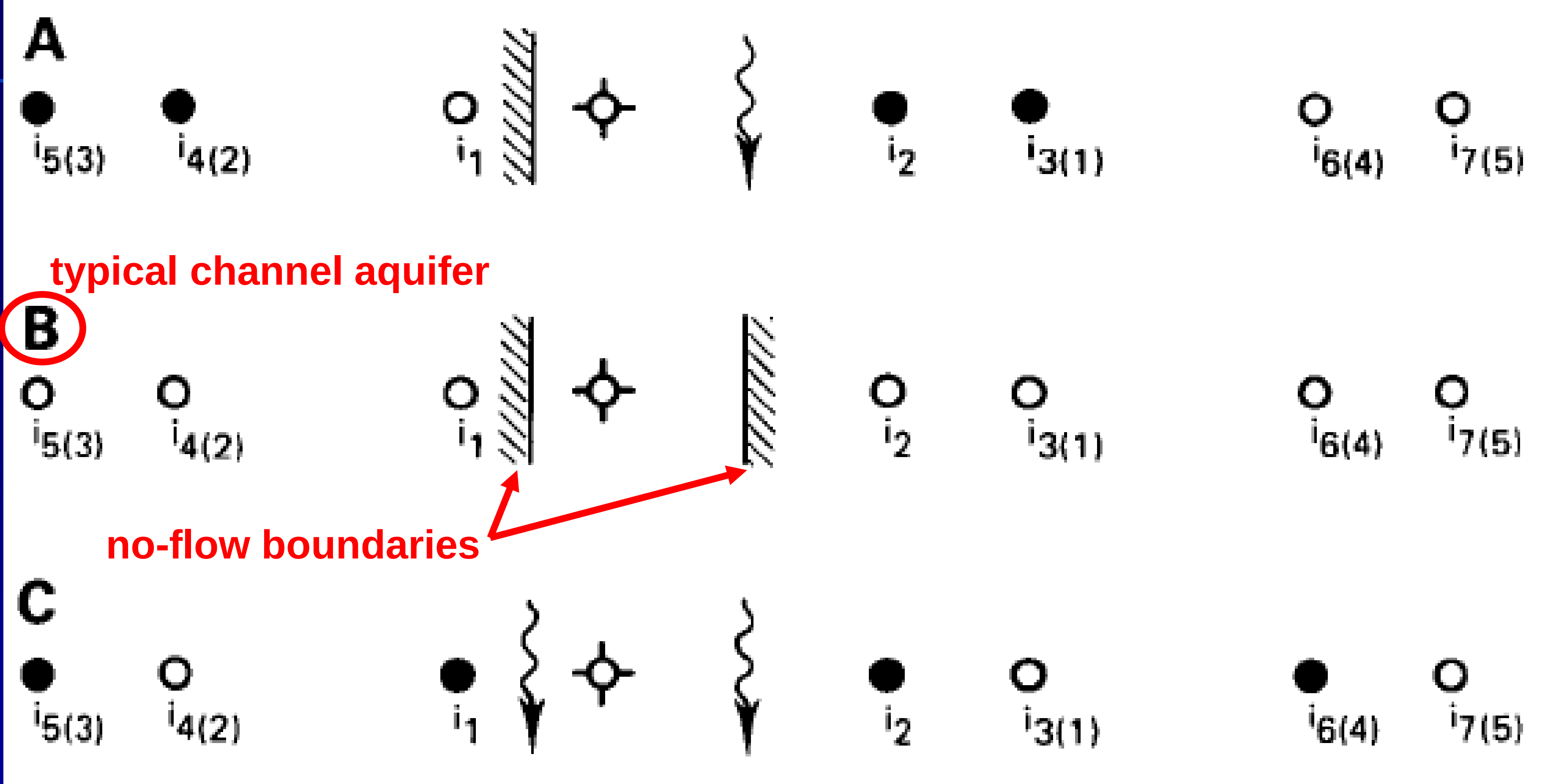
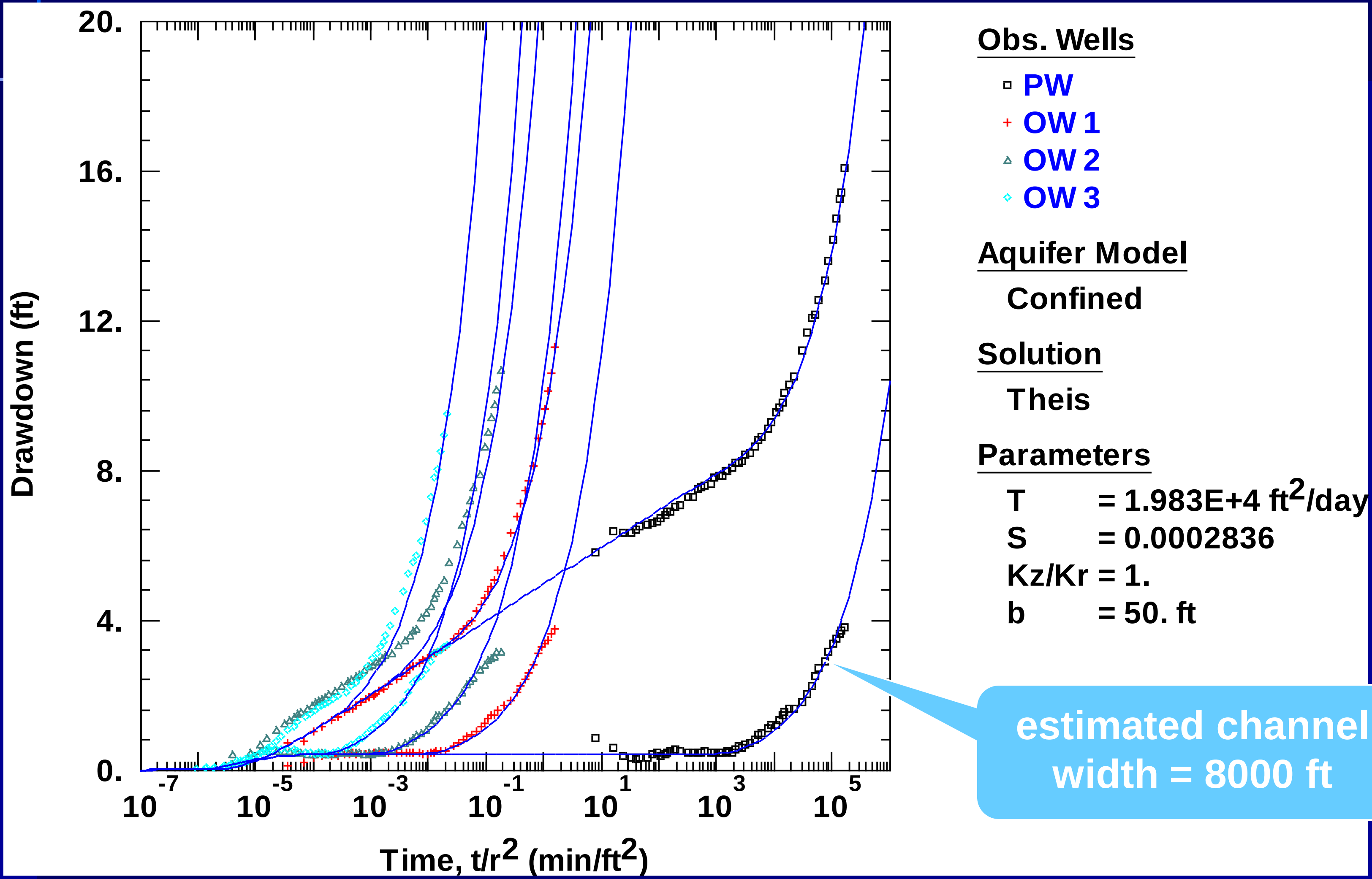


Figure 6.4 Two straight parallel boundaries

from Kruseman and de Ridder (1994)

Theis Analysis w/Channel



Key Concepts and Tips

- **Buried channel aquifer** inferred from late-time derivative response
- Aquifer properties (T and S) estimated efficiently from the infinite-acting period with ***composite plot*** and ***Cooper and Jacob solution***
- ***Channel width*** identified easily by trial-and-error using Theis solution and image wells

Lessons

- Combine derivative analysis with Cooper and Jacob for more reliable estimation of aquifer properties
- Look for infinite-acting radial flow regime to match Cooper and Jacob
- Use derivative analysis to select aquifer models and identify boundaries

Lessons

- When applied carefully, Cooper and Jacob can provide reliable estimates of T and S in confined aquifers with or without leakage
- Do not rely on Cooper and Jacob to determine S from single-well tests due to well loss